

Cosmological parameters

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Abstract. I discuss briefly various bounds on cosmological parameters

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This is a short version of a work presented elsewhere [1].

The universe is believed to be homogeneous and isotropic, and is therefore described by the Friedman-Robertson-Walker (FRW) metric:

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]. \quad (1)$$

Here, r is a dimensionless co-ordinate distance and $a(t)$ is an overall scale parameter. The parameter k determines the curvature of the universe. By adjusting the definition of r , we can make k to have one of these three values: $-1, 0, 1$.

The evolution of the universe depends on the contributions to the energy-momentum from different kinds of sources. It is useful to distinguish between the following kinds:

- non-relativistic matter, like what we are made of;
- ultra-relativistic matter, e.g., photons;
- vacuum energy, which can be seen as a contribution to the cosmological constant.

The second contribution, as indicated by the microwave background temperature, is presumably very small. Ignoring it, we can write down the evolution equations of the universe as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi GT_{\mu\nu} + \Lambda g_{\mu\nu}. \quad (2)$$

Here, $R_{\mu\nu}$ is the Ricci tensor which is defined through the metric of Eq. (1), and $R = R_{\mu\nu}g^{\mu\nu}$. Due to the homogeneity and isotropy, one gets only two independent equations. One of them amounts to the fact that the density of non-relativistic matter, ρ , goes like a^{-3} . The other one is

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{k}{a^2}. \quad (3)$$

The ratio on the left side at the present time is called the Hubble parameter H_0 . For the terms on the right side, one normally introduces the dimensionless parameters

$$\begin{aligned} \Omega_m &\equiv \frac{8\pi G}{3H_0^2}\rho_0, \\ \Omega_\Lambda &\equiv \frac{\Lambda}{3H_0^2}, \\ \Omega_k &\equiv -\frac{k}{a_0^2 H_0^2}, \end{aligned} \quad (4)$$

where the subscript '0' means the value at the present era. Eq. (3) thus implies

$$1 = \Omega_m + \Omega_\Lambda + \Omega_k. \quad (5)$$

So we can take only Ω_m and Ω_Λ to be the independent cosmological parameters, along with H_0 .

It is convenient to use dimensionless variables instead of a and t . We define

$$z \equiv \frac{a_0}{a} - 1, \quad \tau \equiv H_0(t - t_0). \quad (6)$$

Putting $\rho = \rho_0 a_0^3/a^3 = (1+z)^3\rho_0$, we can rewrite Eq. (3) in the following form:

$$d\tau = \frac{dz}{1+z} \frac{1}{\sqrt{(1+z)^2(1+\Omega_m z) - z(2+z)\Omega_\Lambda}}, \quad (7)$$

using Eq. (5) to eliminate Ω_k . This can be numerically integrated for any Ω_m and Ω_Λ to obtain the scale parameter at any time t .

The numerical results for different values of Ω_m and Ω_Λ show a wide variety of possibilities. For $\Omega_\Lambda = 0$, the universe closes and collapses to zero volume in the future if $\Omega_m > 1$. For $\Omega_m < 1$, the universe expands forever. The borderline case of $\Omega_m = 1$ represents a flat universe, i.e., one for which $\Omega_k = 0$.

More generally, when $\Omega_\Lambda \neq 0$, the situation cannot be summarized so easily. One usually calls a universe *open* or *closed* depending on whether Ω_k is positive or negative. On the other hand, a universe can be called *elliptic* if it recollapses in the future, and *hyperbolic* if it is evergrowing. For Ω_Λ and Ω_m both non-zero, we can have open hyperbolic, open elliptic, closed hyperbolic or closed

elliptic universes, depending on the precise values of these two parameters. In addition, there can be solutions where the integral of Eq. (7) diverges. These represent universes without any big bang. The big question is: out of these multitudes of possibilities, how to determine which one is our own universe?

The general (and obvious) strategy is the following. We need to find some observable quantity which depends on the cosmological parameters, determine the value of that observable. That will give us the values for the cosmological parameters. Our original question now shifts to the following one: what is a good observable for this purpose?

One usually relies on the measurement of distances of objects as a function of their redshifts. The physical distance to a certain object can be defined in various ways. For example, the "luminosity distance" ℓ is defined in a way that the apparent luminosity of any object goes like $1/\ell^2$. This is related to the co-ordinate distance by

$$\ell(z) = a_0^2 r(z)/a(z) = (1+z)a_0 r, \quad (8)$$

where a light ray coming towards us satisfies the equation $ds^2 = 0$, i.e.,

$$\frac{dr}{dt} = \frac{\sqrt{1 - kr^2}}{a}, \quad (9)$$

r being the dimensionless co-ordinate distance introduced in Eq. (1). Integration of this equation gives

$$H_0 \ell(z) = \frac{1+z}{\sqrt{|\Omega_k|}} \operatorname{sinn} \left[\sqrt{|\Omega_k|} \int_0^z \frac{dz'}{\sqrt{(1+z')^2(1+\Omega_m z') - z'(2+z')\Omega_\Lambda}} \right]. \quad (10)$$

where "sinn" means the hyperbolic sine function if $\Omega_k > 0$, and the sine function if $\Omega_k < 0$. If $\Omega_k = 0$, the sinn and the Ω_k 's disappear from the expression and we are left only with the integral. Measurement of the apparent magnitudes of type Ia supernovas have recently been used to derive the cosmological constants from this measure of distance.

There are, of course, other ways of determining distances. But, as I said in the beginning, this is only an outline of the full talk. More details, as well as references to the literature, can be obtained in the fuller version [1].

References

1. P B Pal in Proc. of "Discussion meeting on Recent Developments in Neutrino Physics" (hep-ph/9906447) Pramana (1999) (in press)